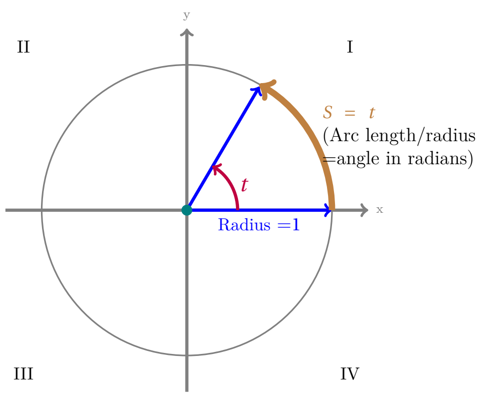
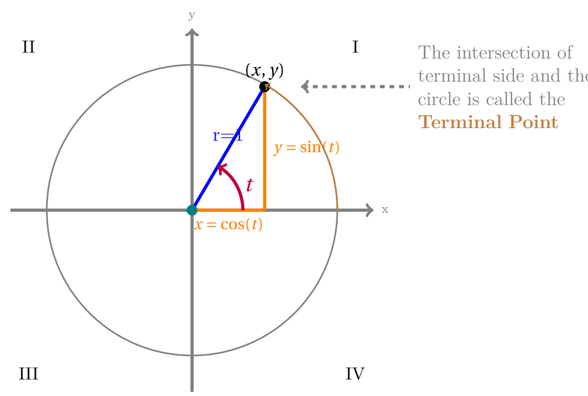
5.2: The Unit Circle and Sine and Cosine Functions

- Equation of a circle centered at (a, b) with radius r is $(x-a)^2 + (y-b)^2 = r^2$
- The unit circle is the circle of radius 1 centered at origin.
- Equation of the **unit circle** is $x^2 + y^2 = 1$ by the distance formula.

Sine and Cosine Functions



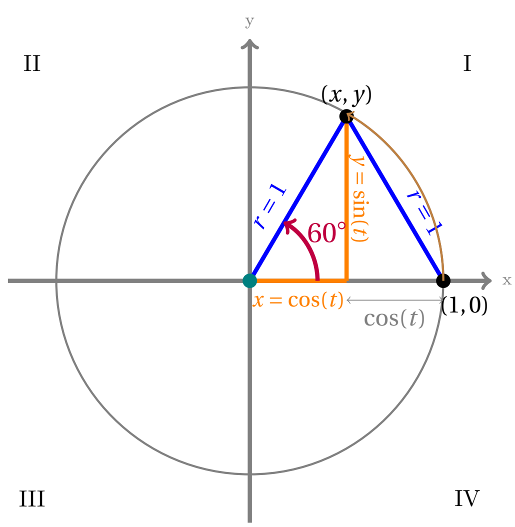


- **Pythagorean Identity:** $\sin^2(t) + \cos^2(t) = 1$ (it is derived from the equation of unit circle).

  Now, you can complete Problems 1-3.

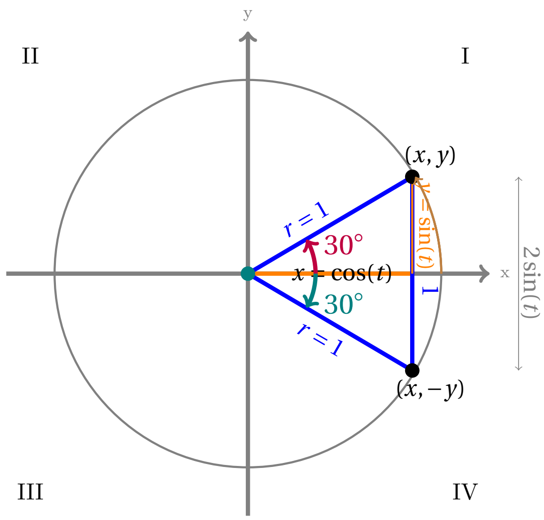
Sine and Cosine of Well-known Angles in the First Quadrant:

We are using the following geometric figures, the properties of equilateral triangles, the properties of isosceles triangles and Pythagorean Theorem to find Sine and Cosine functions for $t = 60°, 30°, 45°$.



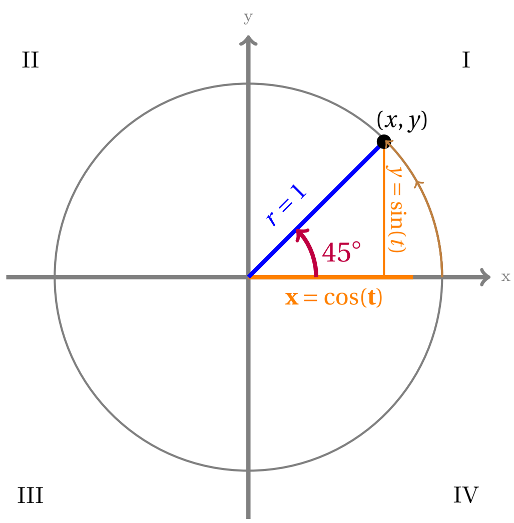
- $t = 60^\circ = \frac{\pi}{3}$ radian: By above picture, $\boxed{\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}}$

  By Pythagorean identity: $\sin^2\left(\frac{\pi}{3}\right) + \frac{1}{2^2} = 1 \implies \sin^2\left(\frac{\pi}{3}\right) = \frac{3}{4} \implies \boxed{\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}}$



- $t = 30^\circ = \frac{\pi}{6}$ radian: As shown in the above figure, $\boxed{\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}}$

  By Pythagorean identity: $\cos^2\left(\frac{\pi}{6}\right) + \frac{1}{2^2} = 1 \implies \cos^2\left(\frac{\pi}{6}\right) = \frac{3}{4} \implies \boxed{\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}}$



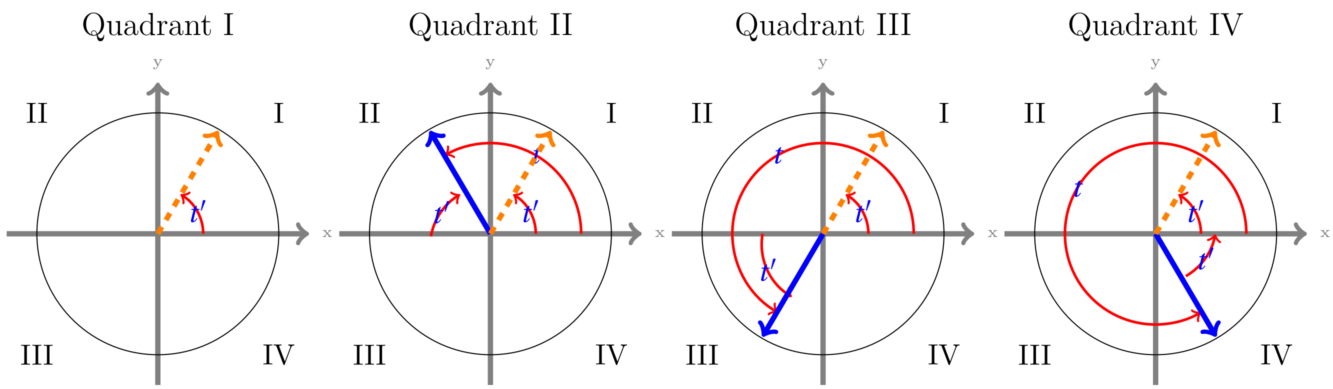
- $t = 45^\circ = \frac{\pi}{4}$ radian: As shown in the above figure, $\sin(\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$.

  By Pythagorean identity: $\cos^2\left(\frac{\pi}{4}\right) + \sin^2\left(\frac{\pi}{4}\right) = 1 \underset{\text{Since they are equal}}{\implies} \cos^2\left(\frac{\pi}{4}\right) = \frac{1}{2} \implies$

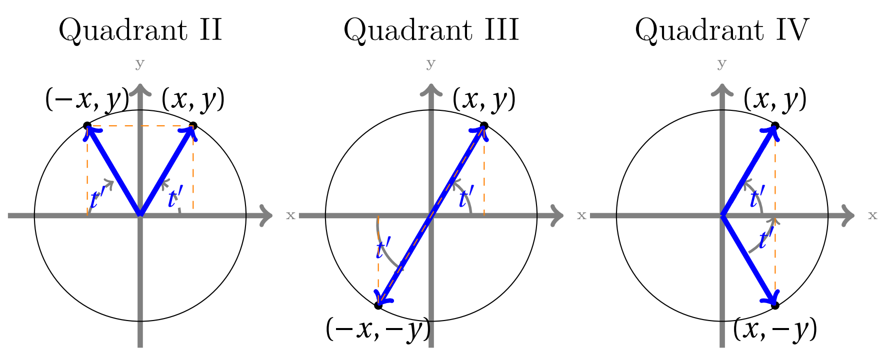
  $\boxed{\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}}$ and $\boxed{\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}}$

- *Now, you can complete Problem 4.*

Reference Angles



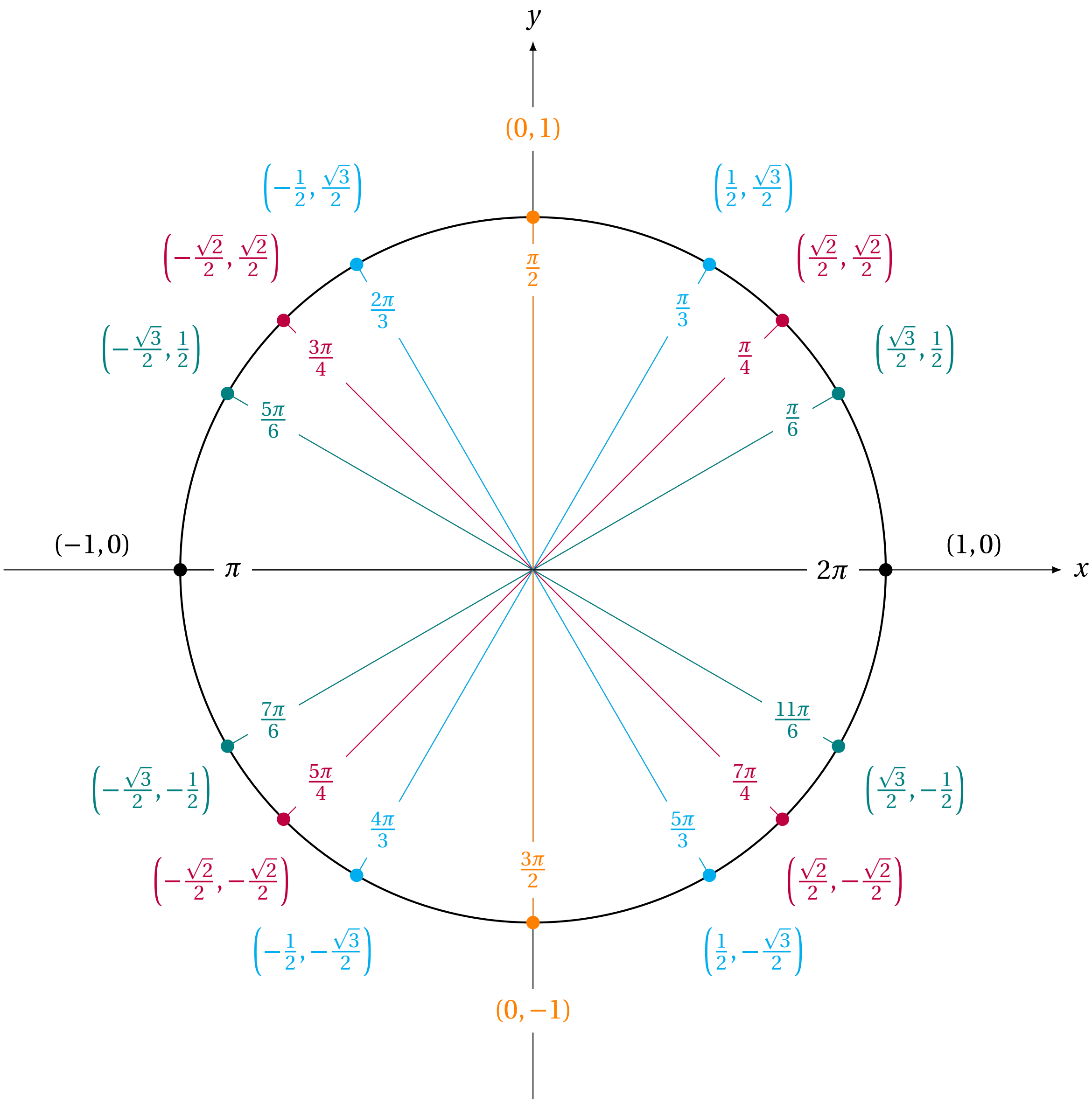
Why Reference Angles Work



- Second Quadrant Reference angle is $\pi - t$ or $180° - t°$. Sine is positive and Cosine is negative.
- Third Quadrant Reference angle is $t - \pi$ or $t° - 180°$.

  Sine and Cosine are both negative.
- Fourth Quadrant Reference angle is $2\pi - t$ or $360° - t°$. Sine is negative and Cosine is positive.
- *Now, you can complete Problems 5 and 6.*

What You Need to Memorize on the Unit Circle



- In The Unit Circle: $\sin(t) = y \quad \cos(t) = x \quad \tan(t) = \frac{y}{x}$ when $x \neq 0$ and etc.
- **Pythagorean identity**: $\cos^2(t) + \sin^2(t) = 1$. Note that we replaced $(\sin(t))^2$ with $\sin^2(t)$ and $(\cos(t))^2$ with $\cos^2(t)$.

1. If $\cos(t) = \dfrac{5}{13}$ and $0 < t < \dfrac{\pi}{2}$, what is $\sin(t)$?

2. If $\sin(t) = \dfrac{4}{5}$and $0 < t < \dfrac{\pi}{2}$, what is $\cos(t)$?

3. Complete the table.

Terminal Point	Quadrant
$\left(-\frac{1}{2}, \quad\right)$	third
$\left(\quad, -\frac{5}{13}\right)$	forth
$\left(\frac{\sqrt{3}}{2}, \quad\right)$	forth
$\left(\quad, -\frac{4}{5}\right)$	third
$\left(\quad, \frac{4}{5}\right)$	second
$\left(-\frac{4}{5}, \quad\right)$	second

4. Memorize the following reference angle information. Explain the pattern that you see. Rewrite the simplified values in the second table.

t	$\sin(t)$	$\cos(t)$
0	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{4}}{2}$
$\frac{\pi}{6}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2}$
$\frac{\pi}{2}$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{0}}{2}$

t	$\sin(t)$	$\cos(t)$
0		
$\frac{\pi}{6}$		
$\frac{\pi}{4}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		

5. Let $t = \frac{5\pi}{6}$. Then the terminal point on the unit circle associated to t is:

(a) $\pi/6$

(b) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

(c) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(d) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

6. Complete the table.

Distance on unit circle	Terminal Point
$\frac{9\pi}{4}$	
$\frac{19\pi}{4}$	
$\frac{29\pi}{4}$	
$\frac{17\pi}{3}$	
$\frac{25\pi}{3}$	

Distance on unit circle	Terminal Point
$\frac{9\pi}{2}$	
$\frac{19\pi}{2}$	
$\frac{29\pi}{2}$	
16π	
25π	

Related Video:

Memorizing Sine and Cosine for Important Angles:
https://mediahub.ku.edu/media/MATH+-+Memorizing+Sine+and+Cosine+for+Important+Angles.m4v/1_ulc6db7m